APPLIED BASIC SURVEY MATH

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Presenter:
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Basic Applied Survey Mathematics

NYSAPLS Conference

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Purpose

- Cover basics of pure and applied mathematics as they pertain to survey
- Areas: algebra, geometry, trigonometry, coordinate math
- [Manual] scientific calculator use
- Basic principles used in survey
- NO discussion of pre-programmed calculators or computers

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What YOU need to do

- Participate (we are all friends here)
- Speak up
- No such thing as “wrong” answer (in this class)
- OR… the only wrong answer is the one you didn’t ask!
- Learning is the ultimate goal
- Use your calculator in class!

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Class overview

- We will travel fast
- Some basic details may be glossed over (though some will be pointed out)
- This is only a small part of the body of knowledge you should know
- Develop skills in researching, reading, trying, applying and learning while you do

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Assessment

- Tear out the quiz sheet at the end of this handout
- Name on answer sheet (1st sheet)
- Take 5-10 minutes
- Bring answer sheet to me when finished
- We will discuss this during the course, so write down your answers on the quiz sheet too

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Quiz time!

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Resources to keep in mind for your future professional development

- Internet
- Colleagues (bosses, peers, others in the profession)
- Societies
- High School
- Community college
- Universities
- Books: read, read, read
- Check out CST program

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Miscellaneous terms & definitions

- Learn the language of math and science
- Acute
- Obtuse
- Complement
- Supplement
- Polygon
- Scalene
- Isosceles…etc.!

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Terms & definitions

- Acute angle is less than a right angle (90°)
- Obtuse angle is greater than a right angle
- The Complement of an angle is determined by subtracting the angle from 90°
- The Supplement of an angle is determined by subtracting it from 180°
- Polygon is an *n*-sided figure with straight sides, of two dimensions, simplest of which is where *n* = 3 (a triangle)
- Scalene triangle has no two of its sides equal
- Isosceles triangle has two sides equal (and as a corollary) two angles equal

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Terms & definitions / 2

- Equilateral triangle has three equal length sides (corollary: also three equal angles, all of which are 60°)
- A Right triangle has a right angle
- Obtuse triangle has an obtuse angle
- Acute triangle has all three angles acute
- Trapezium is a quadrilateral which has no sides parallel
- Trapezoid is a quadrilateral with only two sides parallel
- A Parallelogram has both sets of opposite sides parallel

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Terms & definitions / 3

- Rectangle
- Square
- Rhomboid
- Rhombus
- Similar triangles
- Circle (geometry, trigonometry and surveying)
- Arc
- Chord
- Segment
- Sector
- Tangent
- Secant
- Radian
- π

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Terms & definitions / 4

- Altitude (triangle, prism)
- Directions (bearing, azimuth based from *any* reference direction)
- Coordinates (in two and three dimensions, know differences between notations used in mathematics and surveying)
- Sine (*sin*)
- Cosine (*cos*)
- Tangent (*tan*)
- Cotangent (*cot*)

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Question 5

$$x = \frac{8D}{3(12A + 5B + C)} = \frac{8D}{36A + 15B + 3C}$$

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Calculators

- Functions
- +, -, ×, ÷
- Trig: sin, cos, tan (at least)
- Inverse trig: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ also called (*arc sin*, etc.)
- H.MS (to and from)
- Rectangular ↔ Polar (also *x, y* ↔ *r, θ*)
- Statistical functions (std. dev. *s* or *σ*, average or $\bar{X}$, etc.)

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Calculators / 2

- Memory registers
- Simple programs
- KNOW what the answer will be!!!
- Develop your second guessing skills and second guess your calculator and instrumentation; *this is fundamental to the surveying process!*
- Radians and “small” angle math

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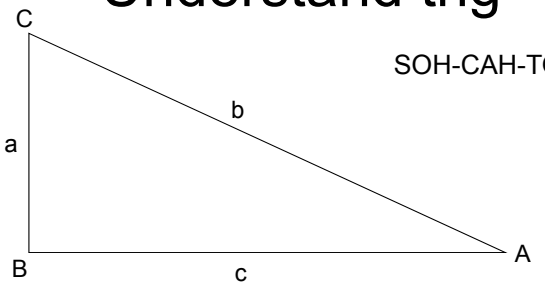
Understand trig



SOH-CAH-TOA

$$\sin = \frac{opposite}{hypotenuse}$$

$$\cos = \frac{adjacent}{hypotenuse}$$

$$\tan = \frac{opposite}{adjacent}$$

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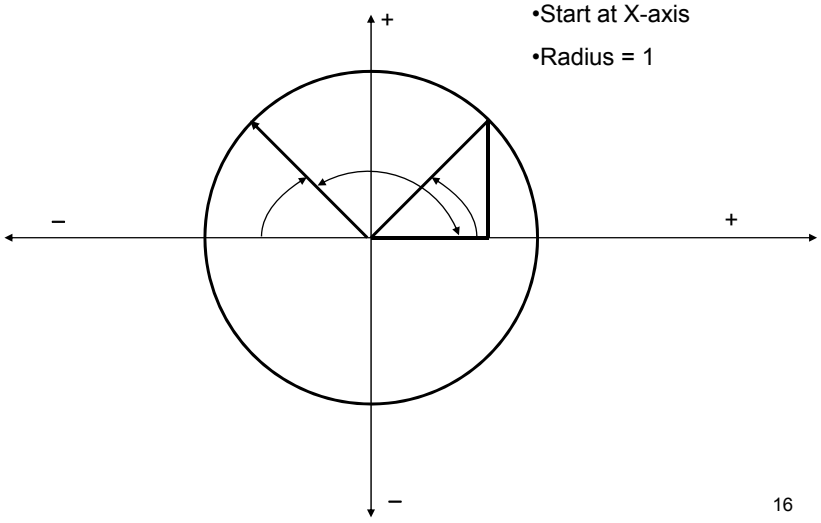
Circle and central angle

- Counterclockwise angle
- Start at X-axis
- Radius = 1



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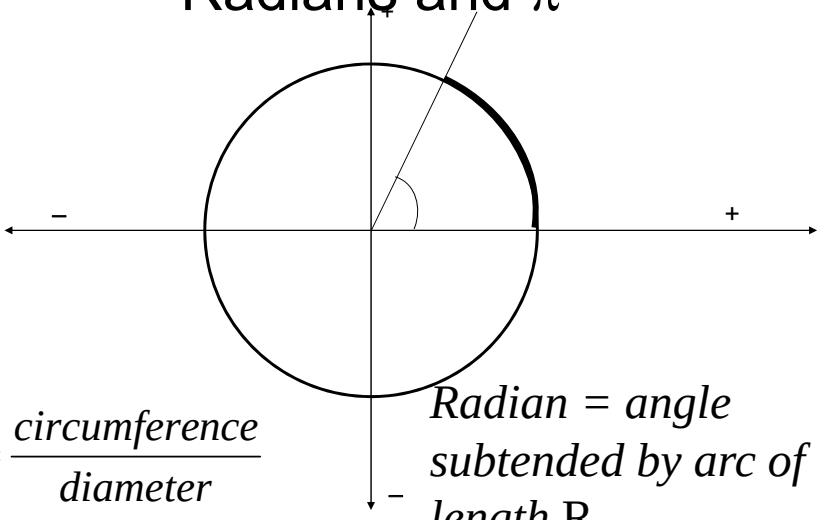
Radians and π



$$\pi = \frac{circumference}{diameter}$$

Radian = angle subtended by arc of length R

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Question 6

- Principle of proportional or similar triangles
- Small triangle is similar to bigger one
- 30 is one-twelfth of 360
- Angles are same; sides of smaller are 1/12 of the bigger
- Therefore Q = 12*7 = 84

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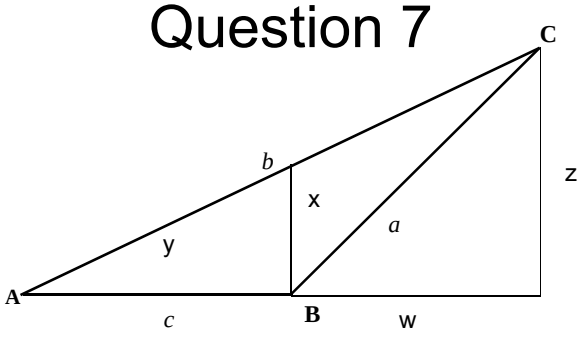
Question 7



$$\sin A = \frac{x}{y}$$

$$\tan B = \frac{z}{w}$$

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The other big functions

- D.MS
- Rectangular $\leftrightarrow$ Polar (also $x, y \leftrightarrow r, \theta$)
- Statistics (std. dev. s or σ, average or $\bar{X}$, etc.)

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D.MS

- Understand how it works
- Don't forget it! [one of the biggest source of mistakes]
- Some calculators support direct addition and subtraction of D.MS formatted numbers
- Know how to do it manually [factor of 60 and 60^2]

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Rectangular ↔ Polar

- Break down a line into its orthogonal components in N-S and E-W directions, i.e. latitudes and departures
- Inverse
- Know how to do this without using this function! [i.e. the theory]
- Theory highly underrated

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Statistics

- Average (mean)
- Standard deviation
- Of 1 or 2 sets of numbers simultaneously
- Various other numbers in storage registers (how many numbers, sum of X's, sum of Y's, etc., etc.)
- Also know how to clear!

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Calculating std. dev. (σ)

$$\sigma = \pm\sqrt{\frac{\sum \nu^2}{n-1}}$$

- ν is the residual of each measurement (measurement – mean)
- $\Sigma\nu^2$ is the sum of the squares of the residuals
- n is the number of measurements
- σ is also referred to as "Root Mean Square Error" (RMSE)

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Simple σ calculation

No.	Measurement	Resi-dual	Resi-dual^2
1	27°43'55"	2	4
2	27°43'55"	2	4
3	27°43'50"	-3	9
4	27°43'52"	-1	1
5	27°44'00"	7	49
6	27°43'49"	-4	16
7	27°43'54"	1	1
8	27°43'56"	3	9
9	27°43'46"	-7	49
10	27°43'51"	-2	4
Mean = 27°43'53"			

$$\sigma = \pm\sqrt{\frac{\sum \nu^2}{n-1}}$$

Sum of ν^2 = 146

n-1 = 9

146/9 = 16.2

Sq. rt. of 16.2 = ±4"

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Calculator warning

- Most handheld calculators do not calculate standard deviation using previously described equation
- Instead a solution that does not require storage of each individual value is used
- This method involves squaring the values

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Calculator warning / 2

- Round-off error, arithmetic overflow and/or arithmetic underflow can occur resulting in erroneous reporting of the standard deviation
- Best handled by only entering the changing parts of the values

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Calculator warning / 3

- If values range between 227°43’27” and 227°43’53”, only enter arc second values
- If values range between 272°34’42” and 227°35’17” drop 227°34’ from them and convert remaining minutes and seconds values to decimal minutes or seconds first
- Etc.

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Learn special things

- Memory registers
- Memory arithmetic
- RPN (if applicable)

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But especially…

- Know what the answer should be!
- Order of magnitude calculations
- Mental arithmetic is essential
- Do not ONLY rely on the calculator

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Traverse calculations

- Balancing angles
- Equal or otherwise?
- Be reasonable, forget 1.79”, 0.0001 ft or 0.000001 Ac
- See example next slide

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Angle balancing example

Sta	Angle	Cumul. Corr.	Rounded Corr	Δ's	Adj Angle
A	100-45-37	2.2”	2”	-2	35”
B	231-23-43	4.4	4	-2	41”
C	17-12-59	6.6	7	-3	56”
D	89-03-28	8.8	9	-2	26”
E	101-34-24	11.0	11	-2	22”
Σ=	540-00-11			-11	00’00”

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Understanding the mathematical assumptions

- Unless using least squares
- What is least squares?
- What assumptions with Compass, Transit and Crandall’s rules?

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Calculate directions

- Use sketches
- Azimuths or
- The hard way (bearings)
- Be able to convert between bearings and azimuths
- Remember that what we call “azimuths” are really North-azimuths
- See example next slides

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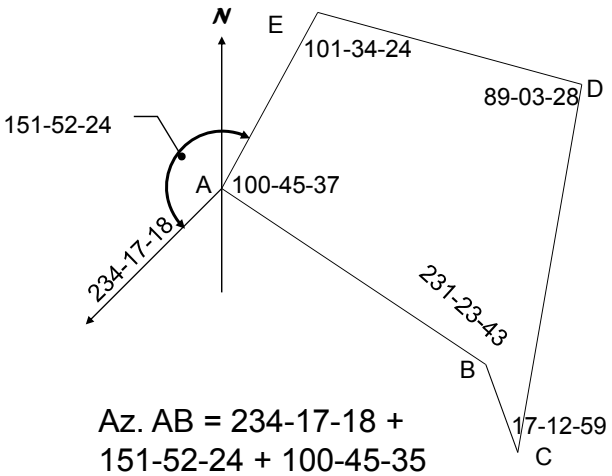
Traverse sketch



Az. AB = 234-17-18 +
151-52-24 + 100-45-35
= <u>126-55-17</u>

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Azimuth computations

- Az. AB = 234-17-18 + 151-52-24 + 100-45-35 = <u>126-55-17</u>
- Az. BC = 126-55-17 + 180-00-00 + 231-23-41 = <u>178-18-58</u>
- Az. CD = 178-18-58 + 180-00-00 +17-12-56 = <u>15-31-54</u>
- Az. DE = 15-31-54 + 180-00-00 + 89-03-26 = <u>284-35-20</u>

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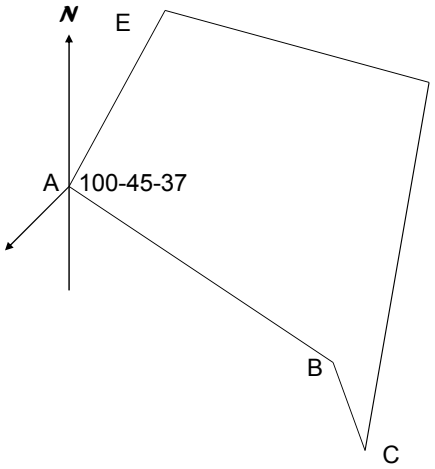
Azimuth computations / 2



- Az. EA = 284-35-20 – 180-00-00 + 101-34-22 = <u>206-09-42</u>
- Using fifth angle to check…
- Az. AB = 206-09-42 – 180-00-00 + 100-45-35 = <u>126-55-17</u>✓

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Know how to describe

- Azimuths
- A mathematical manipulation such as balancing angles, adjusting traverse, etc.

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But don't forget…

- Bearings!
- Angle between 0° and 90° (inclusively) that the subject line makes with the meridian
- *Plus* quadrant designators (N & S before angle and E & W after angle)
- *Exceptions: due north, due east etc.*

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Also know terms

- Latitudes
- Departures
- Which is which?
- Latitudes are north-south
- Departures are east-west
- Don't forget sign "conventions"

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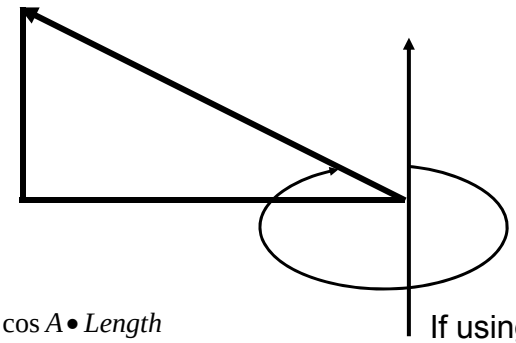
Calculating lat and dep



$$lat = \cos A \bullet Length$$
$$dep = \sin A \bullet Length$$

If using bearings must apply signs manually!!

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Summing lats and deps

- Sums should equal zero if traverse ends where it begins
- If between two control points, the sums should be difference in N- coordinate and E-coordinate values of the control points
- Difference from “shoulds” is your error in latitudes and departures

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Error of closure and precision

- Errors in lats and deps are the orthogonal components of your total error
- Determine total error
- Determine precision [total error divided by perimeter or sum of the traverse lengths]

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Traverse adjustment

- Systematic method of adjusting individual latitudes and departures so that sum equals the “should” value
- Almost anything rational is justifiable in practice, in the exam…that’s different
- Know the conditions and adjust accordingly

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Compass Rule

$$Adj_{lat} = -(E_{lat}) * \frac{linelength}{perimeter}$$

$$Adj_{dep} = -(E_{dep}) * \frac{linelength}{perimeter}$$

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Check adj lat & dep

- Sums should now equal “should” value

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Calculate coordinates

- Use initial value that is given
- Or assume value
- Know how to “translate”
- Understand process of rotation
- Know how to scale [essential for doing state plane coordinates…but other things too, such as “localization”]

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Areas

- Break up into triangles and calculate—NOT!
- Use DMD method or
- Use coordinate method

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Simple area calculation

Point	Northing	Easting		
A	0	0		
B	300	50		
C	450	225		
D	175	550		

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Inversing

- Just the opposite of breaking down a traverse leg
- Uses latitudes and departures [may be obtained by differencing coordinates]
- Pythagorean theorem for length
- $\tan^{-1}$[dep/lat] = azimuth angle (or bearing angle)

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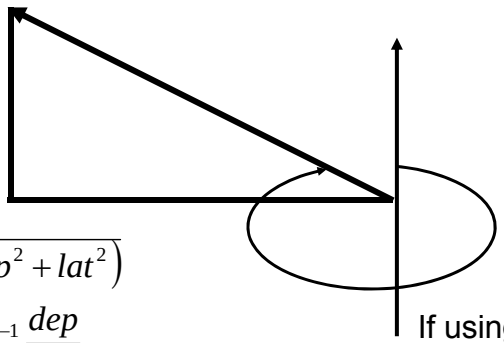
Inversing / 2



$$length = \sqrt{\left(dep^2 + lat^2\right)}$$

$$azimuth = \tan^{-1}\frac{dep}{lat}$$

If using bearings must determine quadrants manually!!

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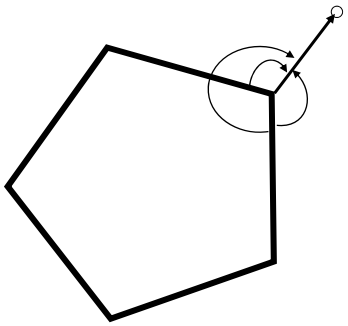
Sideshots



- Simple application of direction calculation
- Then break down into lat and dep
- Add lat to N-coord; dep to E-coord
- No check…unless measured as a sideshot from another traverse point also

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Area by coordinates

Point	Y	X
A	150	75
B	225	150
C	300	125
D	185	0
A	150	75

70,125 - 85,000 = -14,875

Area = 7,437.5

- List coord in order
- Multiply one coordinate along one axis by next coordinate on other axis
- Proceed with this method along one column of coordinates and sum all products
- Then multiply in opposite direction along other column and sum all products
- Difference of the sums is *twice* the area

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Question 8

Point	Lat.	Dep.	Dir.	Dist.	Northing (Y)	Easting (X)
Q					1200	1100
	350	-350				
M					1550	750
	550	450				
N					2100	1200
	81.16	695.28	83°20'30"	700		
O					2181.16	1895.28
P					1650	1600
Q					1200	1100

NP: lat -450;
dep 400
Dist = 602.08;
Azimuth = 138°21'59" or
S41°38'01"E

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Question 8

Northing (Y)	Easting (X)
1200	1100
1550	750
2100	1200
2181.16	1895.28
1650	1600
1200	1100

Area products:

12,044,944 – 10,944,604

Area = 550,170;

if sq. ft, = 12.6 Ac

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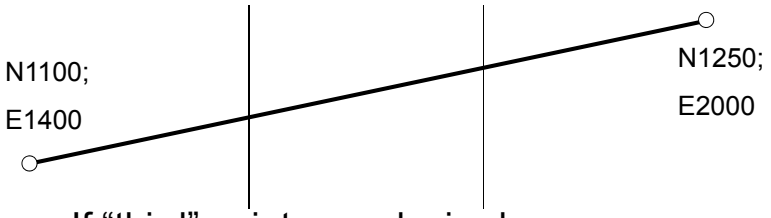
Proportioning w/coordinates



- If “third” points are desired…
- Determine lat and dep
- [lat: 150; dep: 600]
- Take 1/3 [50 and 200]
- Points are at 1150, 1600 and 1200, 1800

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Following slides for reference only

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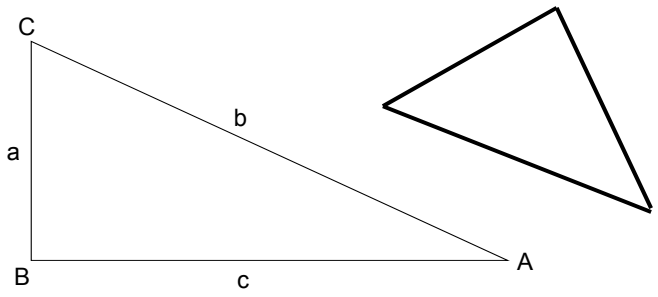
Law of Sines



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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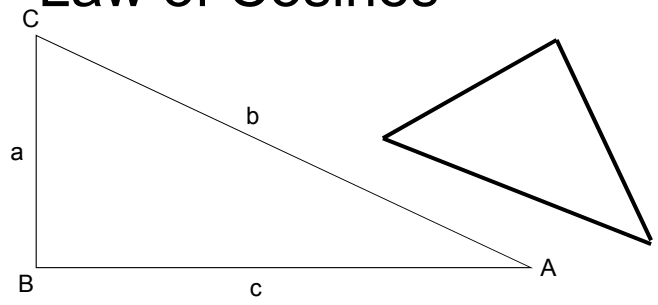
Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos C$$

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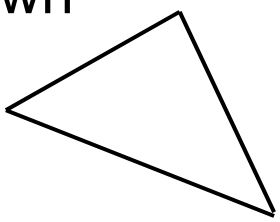
Triangle solution when three sides are known

$$s = \frac{a + b + c}{2}$$



$$\sin \frac{1}{2} A \sqrt{\left[\frac{(s-b)(s-c)}{bc} \right]}$$

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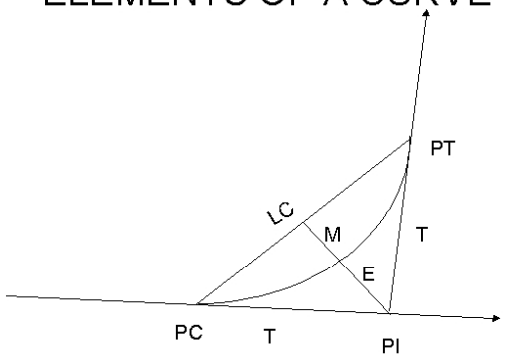
Horizontal curves

ELEMENTS OF A CURVE



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Equations for horizontal curves

$$D = \frac{5729.578}{R}$$ [R can be in any unit, D is in decimal degrees]

$$\frac{\Delta}{L} = \frac{D}{100}$$ [L is length of arc, Δ is central angle of circular curve]

$$T = R\tan\frac{\Delta}{2}$$ [T is length from PC or PT to PI, R is curve radius]

$$L = 100\frac{\Delta}{D}$$

$$LC = 2R\sin\frac{\Delta}{2}$$ [LC is long chord, i.e. distance from PC to PT]

$$stationing PI = PI - T$$ [all in stationing]

$$stationing PT = PC + L$$ [all in stationing]

$$E = T\tan\frac{\Delta}{4}$$ [E is external distance, distance from center of arc to PI]

$$M = E\cos\frac{\Delta}{2}$$ [M is middle ordinate, distance from center of arc to center of long chord]

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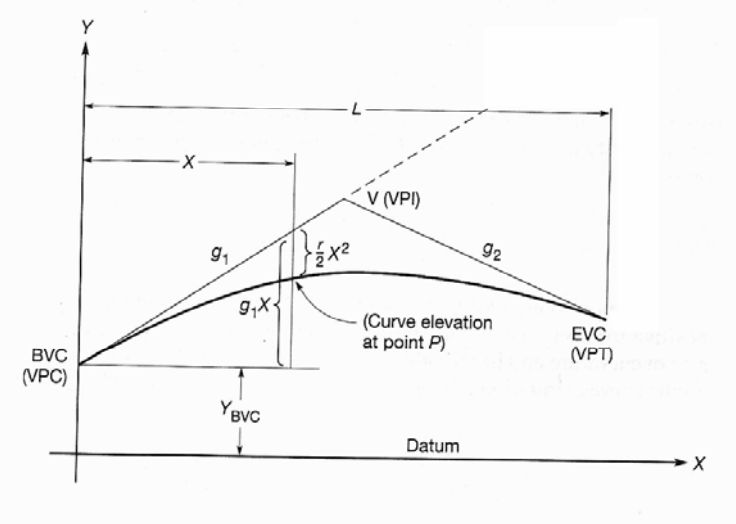
Vertical curves



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Equations for vertical curves

$$Y = Y_{BVC} + g_1 X + \frac{r}{2} X^2$$

Where Y is elevation on curve at any point of interest

$$r = \frac{g_2 - g_1}{L}$$

g_1 and g_2 are approach and departing grades in percent

X is distance in stations from beginning of curve to any point of interest on the curve

L is horizontal length of curve in stations

$$X = -\frac{g_1}{r}$$

Use this eqn. to find hi/lo point

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Questions?

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About the seminar presenter

Joseph V. R. Paiva is a consultant in the field of geomatics and general business, particularly to international developers, manufacturers and distributors of instrumentation and other geomatics tools. Prior to this he was managing director of Spatial Data Research, Inc., a GIS data collection, compilation and software development company. Immediately prior, he was at Trimble Navigation Ltd. His roles included senior scientist and technical advisor for Land Survey research & development, VP of the Land Survey group and director of business development for the Engineering and Construction Division. Previous to that, Paiva was vice president and a founder of Sokkia Technology, Inc., guiding development of GPS- and software-based products for surveying, mapping, measurement and positioning. He has also held senior technical management positions in The Lietz Co. and Sokkia Co. Ltd. Dr. Paiva was assistant professor of civil engineering at the University of Missouri-Columbia, and a partner in a surveying/civil engineering consulting firm. Dr. Paiva's special areas of interest include interface development and design for software and hardware, errors analysis and survey instrumentation of all types. His key contributions in the development field are: design of software flow for the SDR2 and SDR33 Electronic Field Books and the software interface for the Trimble TTS500 total station. He is a member of several professional societies, has presented numerous papers and writes columns for **P.O.B.** magazine and **The Empire State Surveyor**. In May 2006 he completed an approximately three year stint as a columnist for **Civil Engineering News.** He is a Registered Professional Engineer, Registered Land Surveyor, is an ACSM representative to ABET, serving as a program evaluator and team chair on accreditation visits to surveying programs, and has more than 30 years experience working in civil engineering, surveying and mapping. Dr. Paiva is currently working on book to be published soon on the subject of total stations; it is intended to be a practitioner's guide to help in the understanding, operation, testing and adjustment of these ubiquitous instruments.

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