ADVANCED SURVEY MATH

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Presenter:
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Advanced Survey Mathematics

NYSAPLS Conference

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Purpose

- Brief review of traverse calculations, coordinates and areas
- The vector
- Curves
- Intersections (line-line, line-horizontal arc, arc-arc)
- [Manual] scientific calculator use
- Basic principles used in survey
- NO discussion of pre-programmed calculators or computers

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What YOU need to do

- Participate (we are all friends here)
- Speak up
- No such thing as “wrong” answer (in this class)
- OR… the only wrong answer is the one you didn’t ask!
- Learning is the ultimate goal
- Use your calculator in class!

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Class overview

- We will travel fast
- Some basic details may be glossed over (though some will be pointed out)
- This is only a small part of the body of knowledge you should know
- Develop skills in researching, reading, trying, applying and learning while you do

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Resources to keep in mind for your future professional development

- Internet
- Colleagues (bosses, peers, others in the profession)
- Societies
- High School
- Community college
- Universities
- Books: read, read, read
- Check out CST program

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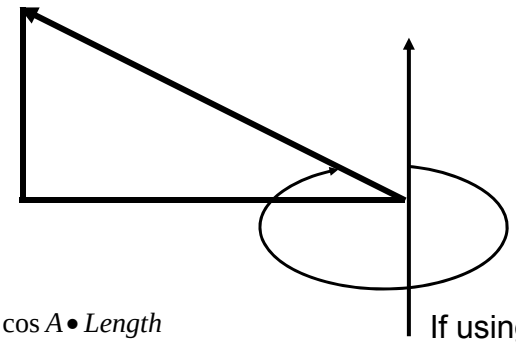
Calculating lat and dep



$$lat = \cos A \bullet Length$$
$$dep = \sin A \bullet Length$$

If using bearings must apply signs manually!!

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Calculate coordinates

- Use initial value that is given
- Or assume value
- Know how to “translate”
- Understand process of rotation
- Know how to scale [essential for doing state plane coordinates…but other things too, such as “localization”]

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Areas

- Break up into triangles and calculate—NOT!
- Use DMD method or
- Use coordinate method

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Simple area calculation

Point	Northing	Easting		
A	0	0		
B	300	50		
C	450	225		
D	175	550		

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Simple area calculation / 2

Line	Latitude	Departure		
AB	300	50		
BC	150	175		
CD	-275	325		
DA	-175	-550		

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Inversing

- Just the opposite of breaking down a traverse leg
- Uses latitudes and departures [may be obtained by differencing coordinates]
- Pythagorean theorem for length
- $\tan^{-1}$[dep/lat] = azimuth angle (or bearing angle)

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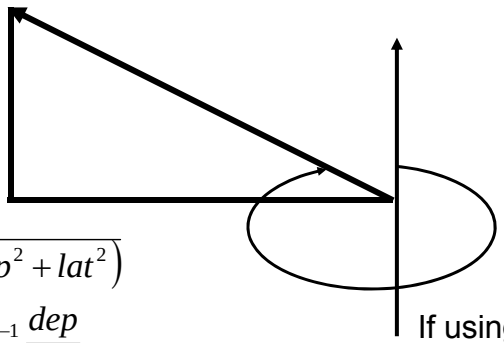
Inversing / 2



$$length = \sqrt{\left(dep^2 + lat^2\right)}$$

$$azimuth = \tan^{-1}\frac{dep}{lat}$$

If using bearings must determine quadrants manually!!

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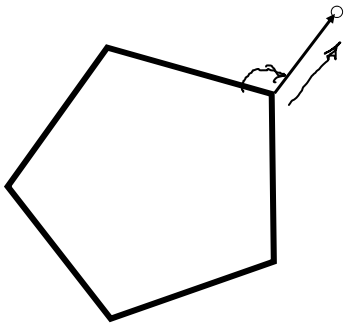
Sideshots



- Simple application of direction calculation
- Then break down into lat and dep
- Add lat to N-coord; dep to E-coord
- No check…unless measured as a sideshot from another traverse point also

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Area by coordinates

Point	Y	X
A	150	75
B	225	150
C	300	125
D	185	0
A	150	75

70,125 - 85,000 = -14,875

Area = 7,437.5

- List coord in order
- Multiply one coordinate along one axis by next coordinate on other axis
- Proceed with this method along one column of coordinates and sum all products
- Then multiply in opposite direction along other column and sum all products
- Difference of the sums is *twice* the area

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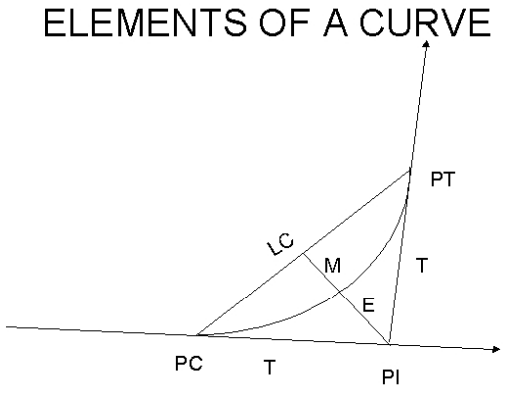
Horizontal curves



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Equations for horizontal curves

$$D = \frac{5729.578}{R}$$ [R can be in any unit, D is in decimal degrees]

$$\frac{\Delta}{L} = \frac{D}{100}$$ [L is length of arc, Δ is central angle of circular curve]

$$T = R\tan\frac{\Delta}{2}$$ [T is length from PC or PT to PI, R is curve radius]

$$L = 100\frac{\Delta}{D}$$

$$LC = 2R\sin\frac{\Delta}{2}$$ [LC is long chord, i.e. distance from PC to PT]

$$stationing PI = PI - T$$ [all in stationing]

$$stationing PT = PC + L$$ [all in stationing]

$$E = T\tan\frac{\Delta}{4}$$ [E is external distance, distance from center of arc to PI]

$$M = E\cos\frac{\Delta}{2}$$ [M is middle ordinate, distance from center of arc to center of long chord]

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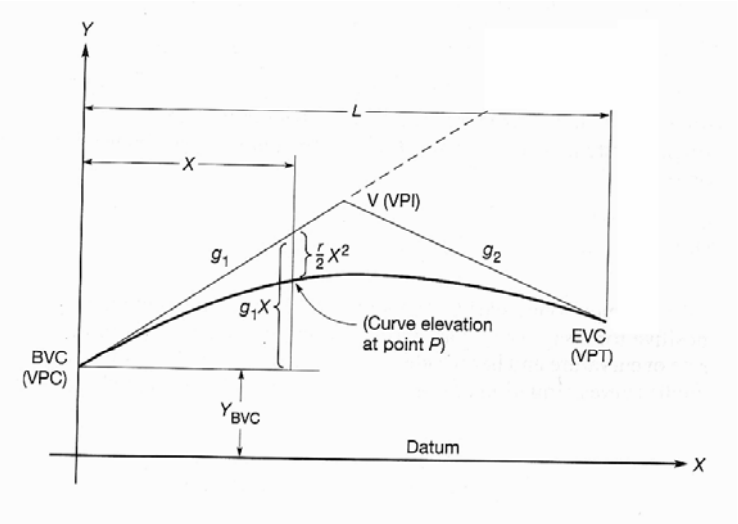
Vertical curves



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Equations for vertical curves

$$Y = Y_{BVC} + g_1X + \frac{r}{2}X^2$$

Where Y is elevation on curve at any point of interest

$$r = \frac{g_2 - g_1}{L}$$

g_1 and g_2 are approach and departing grades in percent

X is distance in stations from beginning of curve to any point of interest on the curve

L is horizontal length of curve in stations

$$X = -\frac{g_1}{r}$$

Use this eqn. to find hi/lo point

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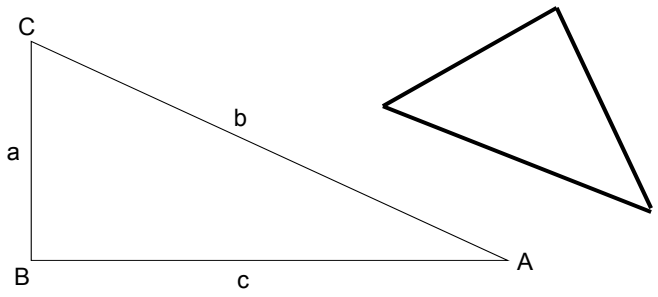
Law of Sines



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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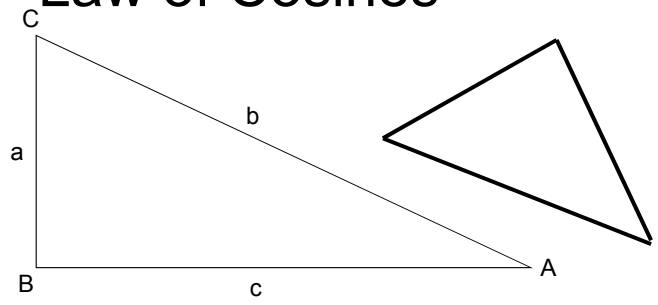
Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos C$$

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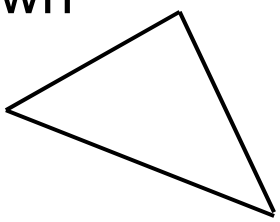
Triangle solution when three sides are known

$$s = \frac{a + b + c}{2}$$



$$\sin \frac{1}{2} A \sqrt{\left[\frac{(s-b)(s-c)}{bc} \right]}$$

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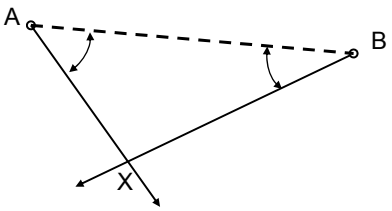
Bearing-bearing intersections



Given: A, B, directions of AX and BX.

Find coordinates of X.

Solution:

1. Calculate length and direction of AB.
2. Calculate angles A and B by subtracting azimuth of AB from azimuth of AX and subtracting azimuth of BX from azimuth of BA. Angle x = 180 – A – B.
3. Use law of sines to solve for either length AX or BX

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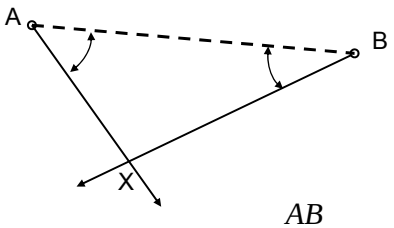
Bearing-bearing solution



$$\frac{AB}{\sin X} = \frac{BX}{\sin A}$$

$$BX = \frac{AB}{\sin X}\sin A$$

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Example for bearing-bearing

- A at N100, E200
- B at N400, E 600
- Az. AX = 135°
- Az. BX = 215°

- Calculate AB
  - Length = 500.00
  - Az. = 53°07’48”

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Example for bearing-bearing

- Calculate X by subtracting azimuths = 80°
- Calculate angle at A
  - 135°00’00” - 53°07’48” = 81°52’12”

$$\frac{AB}{\sin X} = \frac{BX}{\sin A}$$

$$BX = \frac{AB}{\sin X}\sin A$$

$$BX = \frac{500}{\sin 80}\sin 81°52'12'' = 502.611$$

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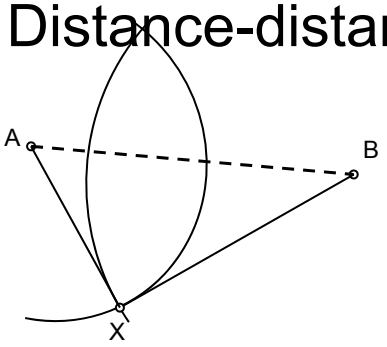
Distance-distance intersections



Given: A, B, distances AX and BX.

Location of X?

Solution:

1. Calculate AB
2. With three sides known, use law of cosines to solve for angle A (or B)
3. By inspection determine which solution is applicable.

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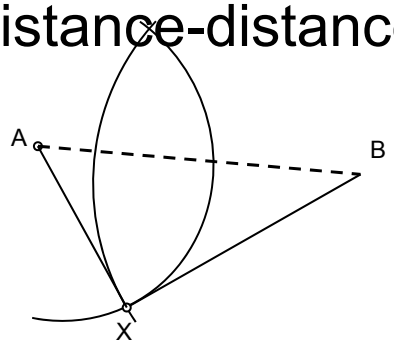
Distance-distance solution



$$BX^2 = AX^2 + AB^2 - 2(AX)(AB)\cos A$$

$$\cos A = -\frac{BX^2 - AX^2 - AB^2}{2 \bullet AX \bullet AB}$$

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Example for distance-distance

- A at N100, E200
- B at N400, E 600
- Distance AX = 600
- Distance BX = 700

- Calculate AB
  - Length = 500.00
  - Az. = 53°07’48”

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Example for distance-distance

$$700^2 = 600^2 + 500^2 - 2(600)(700)\cos A$$

$$\cos A = -\frac{700^2 - 600^2 - 500^2}{2 \bullet 600 \bullet 500}$$

$$A = 78°27'47''$$

$$BX^2 = AX^2 + AB^2 - 2(AX)(AB)\cos A$$

$$\cos A = -\frac{BX^2 - AX^2 - AB^2}{2 \bullet AX \bullet AB}$$

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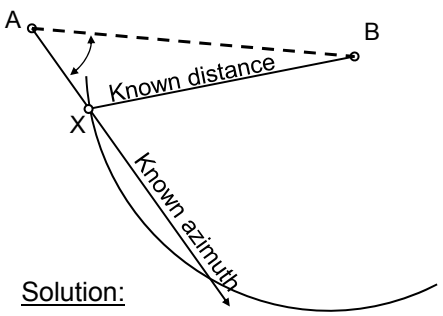
Bearing-distance intersections



Given: A, B, azimuth of AX and length BX.

Where is X?

Solution:

1. Solve for angle A
2. Use law of sines to solve for angle X
3. Two solutions are possible so determine correct solution by inspection

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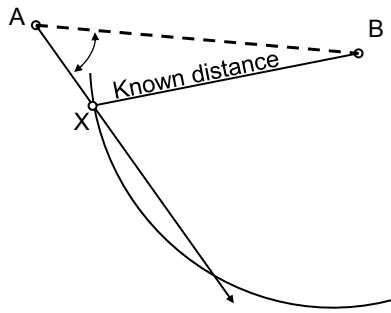
Bearing-distance solution



$$\frac{AB}{\sin X} = \frac{BX}{\sin A}$$

$$\sin X = \frac{AB}{BX} \sin A$$

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Example for bearing-distance

- A at N100, E200
- B at N400, E 600
- Az. AX = 135°
- Distance BX = 600

- Calculate AB
  - Length = 500.00
  - Az. = 53°07’48”

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Example for bearing-bearing

Calculate angle at A

– 135°00’00” - 53°07’48”
= 81°52’12”

$$\frac{AB}{\sin X} = \frac{BX}{\sin A}$$

$$\sin X = \frac{AB}{BX} \sin A$$

$$\sin X = \frac{500}{600} \sin 81°52'12'' = 0.824958$$

$$X = 55°35'03''$$

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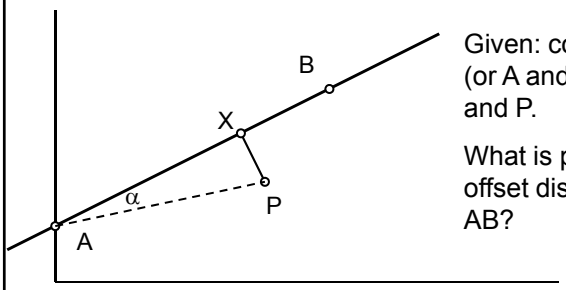
Perpendicular offset



Given: coordinates of A, B, (or A and azimuth of AB) and P.

What is perpendicular offset distance from P to AB?

Solution:

1. Calculate azimuth of AB (if needed)
2. Calculate azimuth and length of line AP
3. Calculate α
4. Now solve right triangle for distance PX

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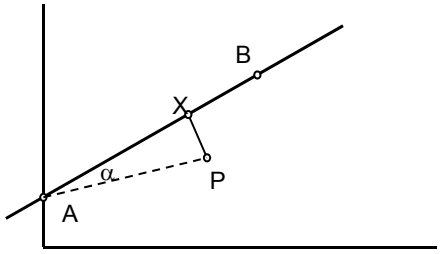
Example for perpendicular offset

- P at N510, E1120
- A at N415, E865
- B at N670, E1550



- Calculate AP from coordinates
- ΔE = 255
- ΔN = 95
- Length = 272.10
- Az. = 69°34’02”

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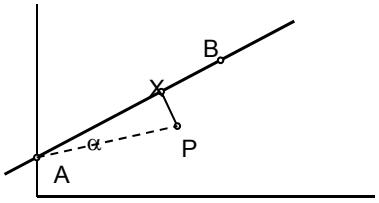
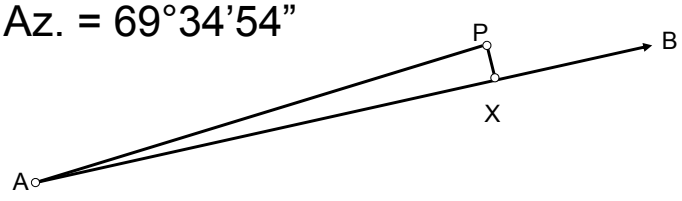
Example for perpendicular offset / 2

- Calculate AB from coordinates
- ΔE = 685
- ΔN = 255
- Length = 730.925
- Az. = 69°34’54”





Note sketch is revised!

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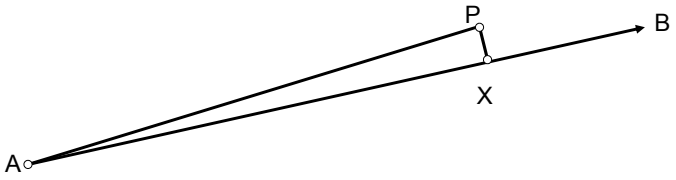
Example for perpendicular offset / 3

- α = 0°00’52”
- PX = AP *sin*α = 0.068



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Questions?

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About the seminar presenter

Joseph V. R. Paiva is a consultant in the field of geomatics and general business, particularly to international developers, manufacturers and distributors of instrumentation and other geomatics tools. Prior to this he was managing director of Spatial Data Research, Inc., a GIS data collection, compilation and software development company. Immediately prior, he was at Trimble Navigation Ltd. His roles included senior scientist and technical advisor for Land Survey research & development, VP of the Land Survey group and director of business development for the Engineering and Construction Division. Previous to that, Paiva was vice president and a founder of Sokkia Technology, Inc., guiding development of GPS- and software-based products for surveying, mapping, measurement and positioning. He has also held senior technical management positions in The Lietz Co. and Sokkia Co. Ltd. Dr. Paiva was assistant professor of civil engineering at the University of Missouri-Columbia, and a partner in a surveying/civil engineering consulting firm. Dr. Paiva's special areas of interest include interface development and design for software and hardware, errors analysis and survey instrumentation of all types. His key contributions in the development field are: design of software flow for the SDR2 and SDR33 Electronic Field Books and the software interface for the Trimble TTS500 total station. He is a member of several professional societies, has presented numerous papers and writes columns for **P.O.B.** magazine and **The Empire State Surveyor**. In May 2006 he completed an approximately three year stint as a columnist for **Civil Engineering News.** He is a Registered Professional Engineer, Registered Land Surveyor, is an ACSM representative to ABET, serving as a program evaluator and team chair on accreditation visits to surveying programs, and has more than 30 years experience working in civil engineering, surveying and mapping. Dr. Paiva is currently working on book to be published soon on the subject of total stations; it is intended to be a practitioner's guide to help in the understanding, operation, testing and adjustment of these ubiquitous instruments.

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